

Tutorial 6

Wavelet Analysis

The **Wavelet Analysis** modules opens a new window when the user clicks on the corresponding button of the left hand side panel of INA. This window is illustrated in Figure 1.

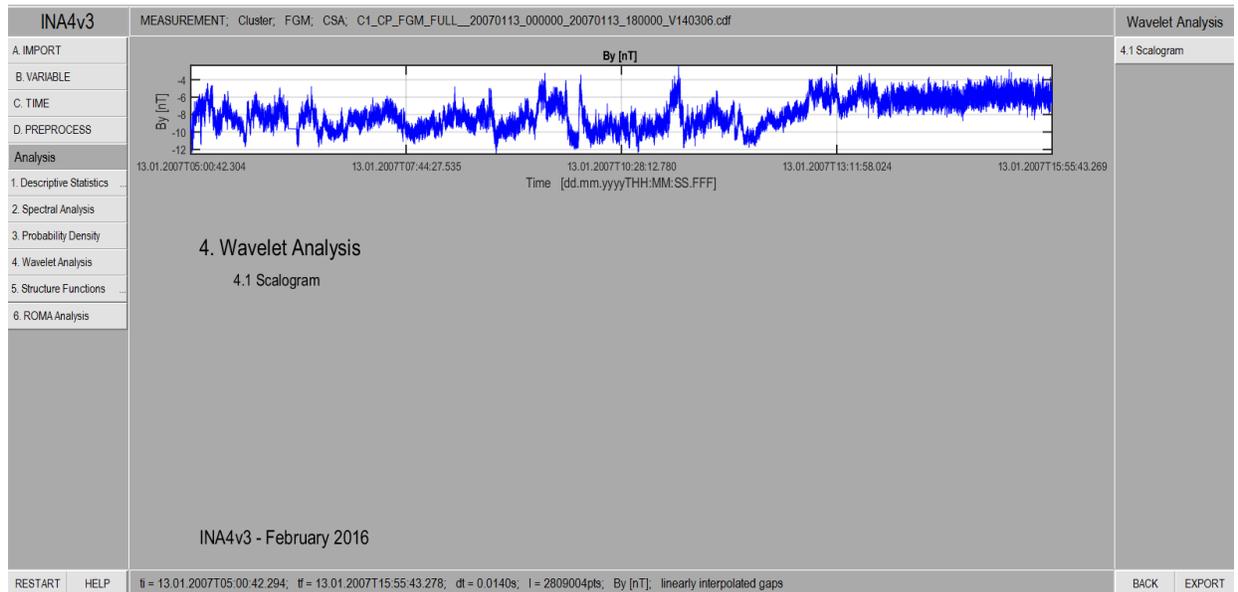


Figure 1. The main page of the Wavelet Analysis tool of INA. The plot shows the time series to be analyzed with this package.

The button at the left hand side of the page illustrated in Figure 1 opens the main wavelet analysis window, illustrated in Figure 2. The module contains 2 subclasses of methods: a) **Scalogram**, and b) **LIM** (Local Intermittency Measure).

The wavelet scalogram illustrated in Figure 2 is the 3D color representation of the wavelet coefficients, where the horizontal axis represents the time t , the vertical axis represents the scale a , and the color scale (z axis) represents the logarithm of the squared modulus of the wavelet coefficients:

$$S(a, t) = |W(a, t)|^2$$

where $W(a, t)$ is the wavelet coefficient matrix, computed using the MATLAB `cwt` function.

The Local Intermittency Measure (LIM) is defined from the wavelet representation as a normalized distribution defined as:

$$LIM(a, t) = \frac{S(a, t)}{\langle S(a, t) \rangle_t}$$

, where $\langle S(a, t) \rangle_t$, is the time average of the scalogram for the scale a . LIM gives the ratio between a certain wavelet coefficient associated to a time t and scale a and the time average of the coefficients belonging to the same scale a . It is computed when the user clicks on the corresponding button on the right hand side panel.

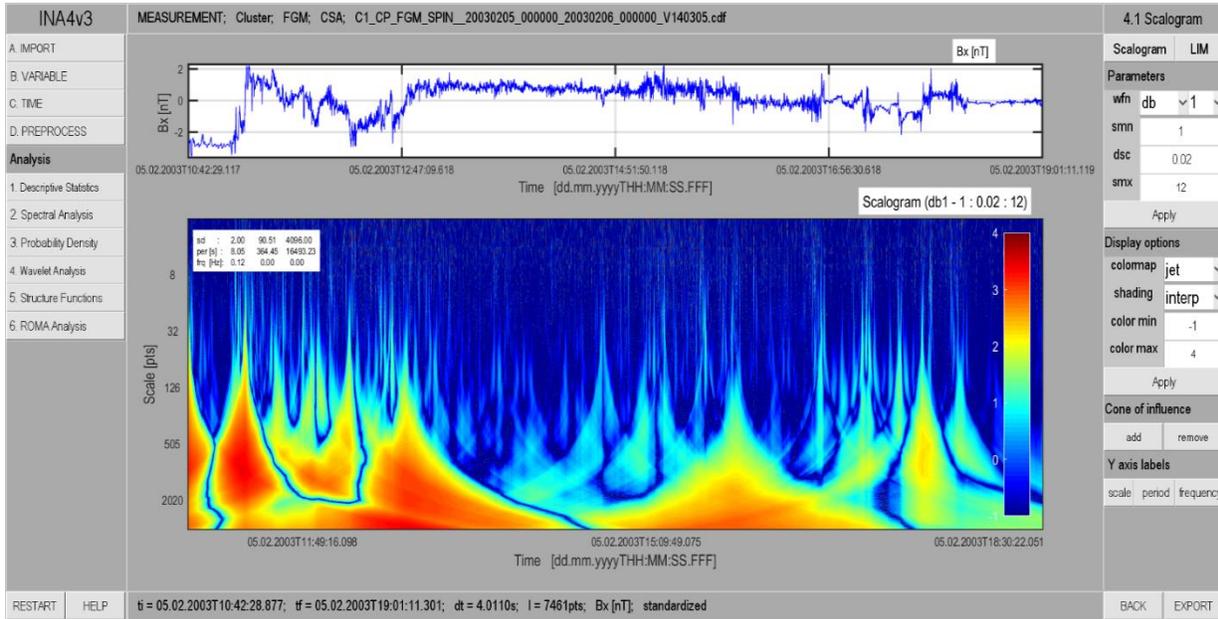


Figure 2. Illustration of the Wavelet Analysis class. The main plot illustrates the wavelet representation of the signal - the scalogram. An explanation of the controlling functions available from the right hand side panel is given in the text below.

The wavelet analysis can be customized by the user in many ways. Two drop-down menus denoted by **wfn** were designed for the selection of the applied mother wavelet of the transformation. The first menu is used to set the wavelet family, and the second one sets the number of vanishing moments. As a rule of thumb, wavelets with a small number of vanishing moments have a better time resolution and can be used to detect abrupt changes in the time domain, like discontinuities, while wavelets with a large number of vanishing moments have a better frequency resolution, and can be used to detect periodicities in the time series. INA offers three wavelet families:

- (i) **db** – are the Daubechies’ extremal phase wavelets, with db1 being identical to the Haar wavelet,
- (ii) **sym** – the Symlets family also known as Daubechies' least-asymmetric wavelets
- (iii) **coif** - the Coiflets family of wavelets.

db and **sym** can be used with up to 10 vanishing moments, while **coif** can only have a maximum of 5.

The scale parameter of the wavelet transform varies as 2^s . By default s ranges between 0 and the smaller integer to $\log_2(N)$, where N is the length of the signal. To zoom a certain scale range of the scalogram, the default limits of s can be changed in the fields denoted by **smn** and **smx**. The field denoted by **dsc** is for the selection of the optimal increment between consecutive s values. For investigating low and high details of the signal in terms of scale, **dsc** should be selected respectively to high and small values.

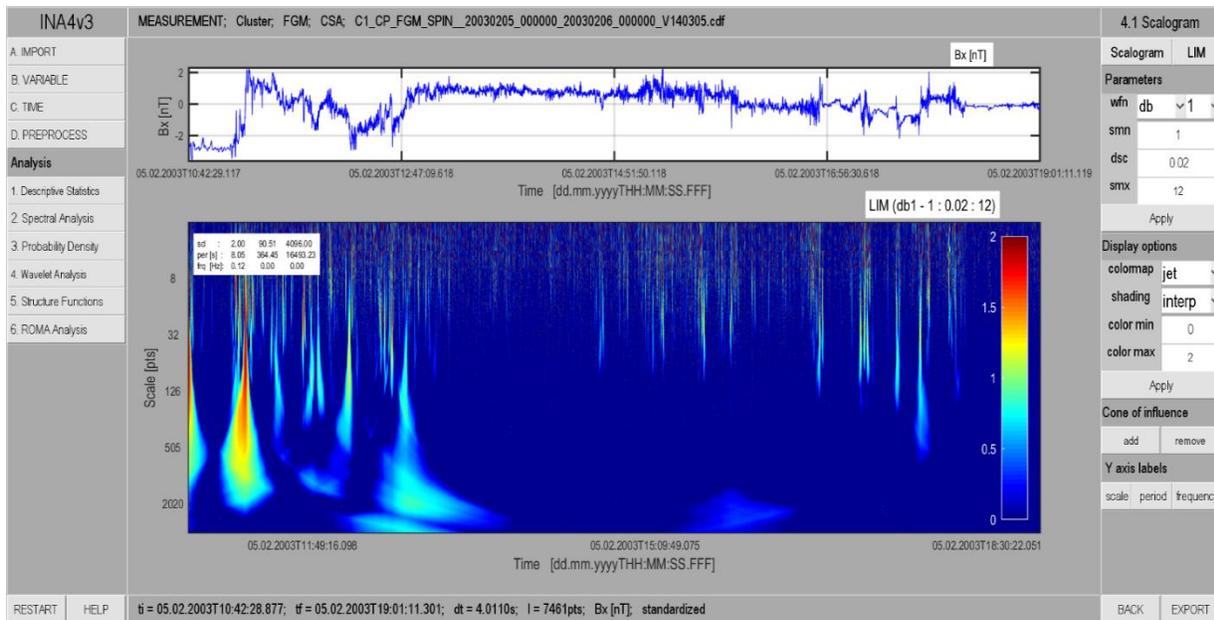


Figure 3. Illustration of the Wavelet Analysis class. The main plot illustrates the LIM representation of the signal.

There are four adjustable input fields in INA that control the wavelet representation of the results. The color scale range of both, scalogram and LIM plots, can be set in the fields denoted by **cmn** and **cmx**. The **flat**, **interp** and **faceted** options are for switching between rough and interpolated representations of the scalogram and LIM plots. In case of using the **flat** option the color coded coefficients appear in parcels whose dimensions directly represent the resolution of the transformation at the given scale and time. The user can also change the default color map, which controls the color palette of an image, by choosing a different option from the corresponding drop-down menu. The display option denoted by **add** and **remove** enables the display of a boundary in the scalogram plot called “Cone of influence” beyond which (towards the scalogram edges) the wavelet coefficients are inaccurate because of the finite-length of the input time-series.

The buttons at the bottom of the right hand side control panel, control how the vertical axis of the scalogram is represented and can switch between **scale** (which gives the scale in number of points), **period** (which gives the scale in seconds) and **frequency** (which gives the scale in Hertz). Although there is no direct relationship between the wavelet scale and frequency,

a set of pseudo-frequencies corresponding to the set of scales and the specific wavelet function used in the transform is computed using the MATLAB `scal2frq` function. With these pseudo-frequencies we can also estimate the period, as $1/\text{frequency}$.

Figures 2 and 3 depict the Scalogram and LIM representations of the same time series described in the previous tutorials. The central panel shows the time series (top) and the analysis results (bottom), the wavelet Scalogram (in figure 2) and its LIM representation (in figure 3). The results are obtained with: `db1` (Haar), as the wavelet function and scales a between 2^1 and 2^{12} . The display parameters used here are: a jet color map, an interpolated shading and the color limits of `[-1 4]` for Scalogram and `[0 2]` for LIM.